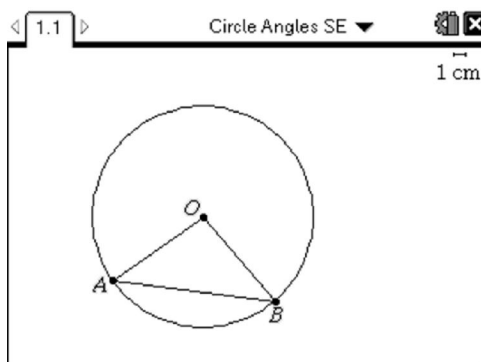


S3 Self-Exploration of Circle Angles using Nspires

Triangle on a Chord and Centre

1. Start a new document with a Geometry application.
2. Save it under the name 'Circle Angles SE'
3. Draw a circle
4. Draw a triangle with vertices on the centre and the circumference of the circle.
5. Label the centre O, and the points on the circumference as A and B.....

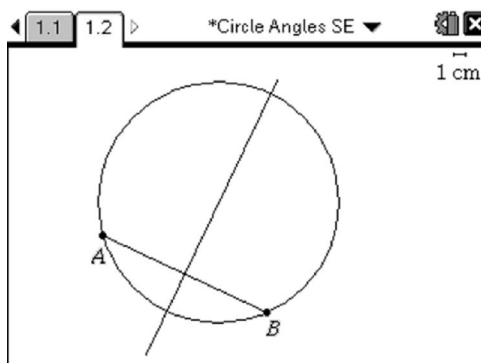
[The line segment AB is known as a **chord**. It joins two points on the circumference of a circle]



6. Now measure angles OAB and OBA.
7. Move the points A and B around the circumference.
8. *What do you notice?*
Can you explain why this happens?
Write down your explanation in your jotter.

Perpendicular Bisector of Chords

1. Insert a new page with a Geometry application.
2. Draw a circle.
3. Hide the centre point of the circle.
4. Draw the chord AB.
5. Construct the perpendicular bisector to the chord AB.
6. Grab and drag the end of the perpendicular bisector, so that it reaches to the other side of the circle, like so:



7. Now draw another, new chord CD along with its perpendicular bisector. Extend this second perpendicular bisector, as you did in step 6.
8. Move the points A, B, C and D around.

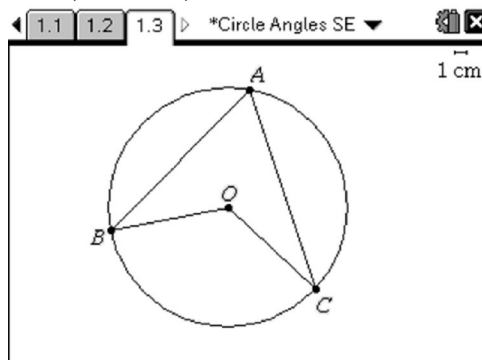
What do you notice?

Can you explain why this happens?

Write down your explanation in your jotter.

Inscribed Angle Theorem

1. Insert a new page with a Geometry application.
2. Draw a circle
3. Draw a 4 sided polygon with 3 vertices on the circumference and one vertex on the centre.
4. Label the centre O and the vertices A, B and C, as shown:



5. Measure the angle BOC.
6. Now move points A, B and C around.
7. You ought to find that sometimes the angle at BOC is on the interior of the polygon, and sometimes it is on the exterior. This is because the Nspire software has been programmed to always measure the smallest angle. From now on, please focus on the exterior angle BOC, even if it's not the one displayed and being measured.
8. Measure the angle BAC. This is called the **inscribed angle**, as it's where two chords meet on the circumference.
9. Now move point B or C and look for a connection between and BAC and exterior angle BOC.
10. *What do you notice?*
Can you prove why this happens, using algebra?

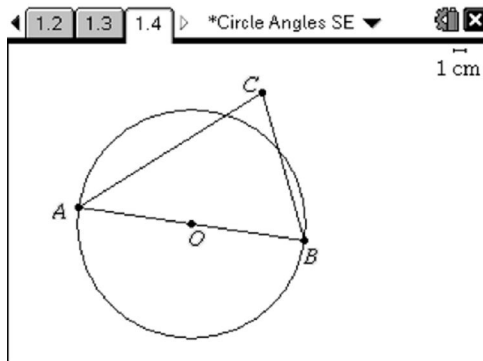
Helpful hint for your proof: *Sketch the diagram nice and large in your jotter.*
Draw a line through the circle passing through A and O.
Label angle BAO to be x, and angle CAO to be y.
Now fill in as many other angles as you can, in terms of x and y.

11. Now, copy and complete the following sentence:
"The inscribed angle is _____ the angle subtended at the centre"
12. Now move point A.
What changes?
What doesn't change?
Does this surprise you at all?
13. Now move point B or C so that BOC is 180° . This makes BC become a diameter.
What sort of triangle has ABC now become?
14. Now move point A. You should find that your triangle keeps the same property.
What sort of maths might you use on such a triangle?

The next activity approaches this property from a different angle. Different *angle*.... Geddit?!?

Angle in a Semi-Circle

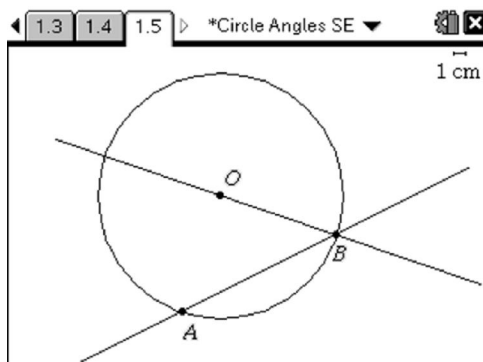
1. Insert a new page with a Geometry application.
2. Draw a segment, and label its endpoints A and B
3. Construct the midpoint of the segment, and label it O.
4. Construct a circle, with centre O and radius either OA or OB.
5. You should be able to grab either A or B and move it around, and AB remains as a diameter of the circle.
Check now that it does behave in this way.
6. Construct a triangle using diameter AB and a third point C, positioned somewhere outside the circle:



7. Measure angle ACB.
8. Before you move point C around, think about what the angle ACB will do when you move point C towards the inside of the circle.
9. Now move point C and see if you were right.
10. *How does this observation compare to the result from the Inscribed Angle Theorem?*

Tangent to a Circle

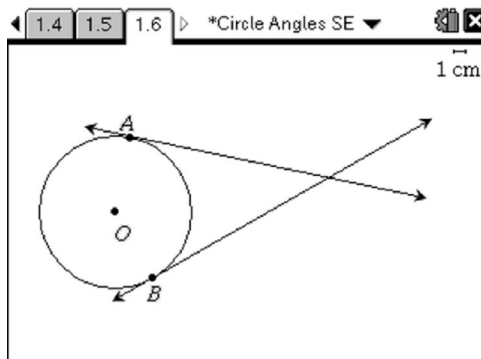
1. Insert a new page with a Geometry application.
2. Draw a circle, and label the centre O.
3. Use the line tool (not the segment tool) to draw a chord AB that extends outside the circle. Grab the ends of the line to extend it further if required.
[A chord that extends outside of the circle is called a **secant**.]
4. Use the line tool again to connect O to B, as shown:



5. Measure angle OBA.
 6. Now move point B towards point A, around the circumference of the circle.
What do you notice happens to the angle OBA when B reaches A?
What size angle would you expect it to display?
[A secant that intersects a circle in only one place is called a **tangent**.]
 7. The Nspire Geometry software only measures angles to a certain degree of accuracy. The construction that you have just made pushes the limits of that accuracy, so it's unlike to show the angle that it would *really* be in an exact geometrical construction.
Can you now construct and write down an explanation, or mathematical argument, to justify the size of angle that it ought to display?
-

The Tangent Kite

1. Insert a new page with a Geometry application.
2. Draw a circle, and label the centre O.
3. Use the tangent tool to draw a tangent to the circle at a point on its circumference. Move your pointer over the tangent icon in the top left corner to read about how to use it.
4. Grab the end of tangent line and extend it to the side.
5. Repeat steps 3 and 4 to draw a second tangent, which intersects the first, as shown:



6. Label the centre as O, and the tangent points to be A and B.
7. Place a point at the intersection of the two tangents and label it P.
8. Draw two segments from O to A, and O to B.
9. You should already know the sizes of angles OAP and OBP, so measure angle AOB and angle APB.
10. Move A or B, and notice what happens to the angles AOB and APB.
11. *What can you say about angles AOB and APB?*
Can you prove why this happens, using algebra?

Helpful hint for your proof:

Sketch the diagram nice and large in your jotter.

Draw a line joining O to P

Label angle AOP to be x

Now fill in as many other angles as you can, in terms of x .

12. Now consider the lengths of AP and BP.

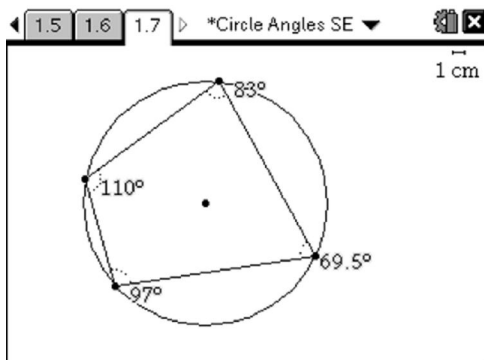
Are they the same length?

You can check to see if they are by using the measure length tool.

Can you write down an explanation for what you have discovered about their lengths, and why it must be true?

The Cyclic Quadrilateral

1. Insert a new page with a Geometry application.
2. Draw a circle.
3. Draw a 4 sided polygon inside the circle, with each vertex on the circumference.



4. Measure each of the 4 interior angles of the quadrilateral.

5. Move the vertices around.

Can you discover any property about the angles?

Can you prove your discovery, using algebra?