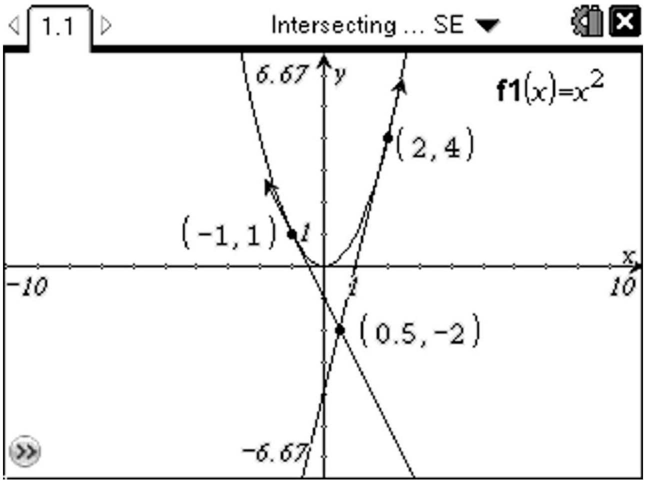
Intersecting Tangents of a Parabola

This activity will take you through the construction that is required to investigate a property of the intersection point of two tangents to a parabolic curve.

You will be required to document your results and discoveries by writing them out, on paper.

We are going to create a construction on the TI-Nspire that looks like this:



This shows a quadratic function f(x)=x^2, with two tangents drawn in, from the points (-1,1) and (2,4)

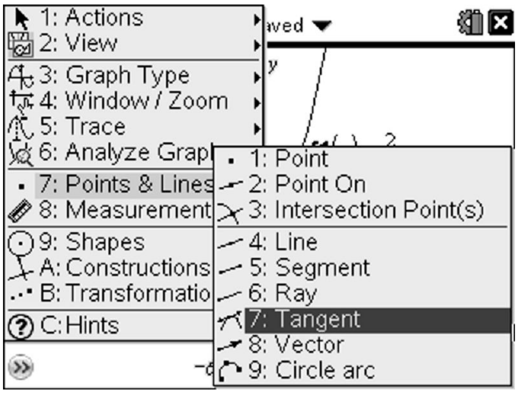
The tangents intersect at the point (0.5, -2)

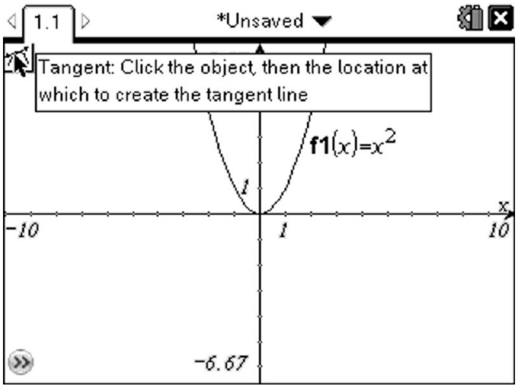
Task 1

There is a connection between the points on the quadratic and the intersection point of the tangent lines.
You have to discover what that connection is.
You should aim to be able to predict the coordinates of the intersection point of the tangents from only knowing the coordinates of the points on the parabola.

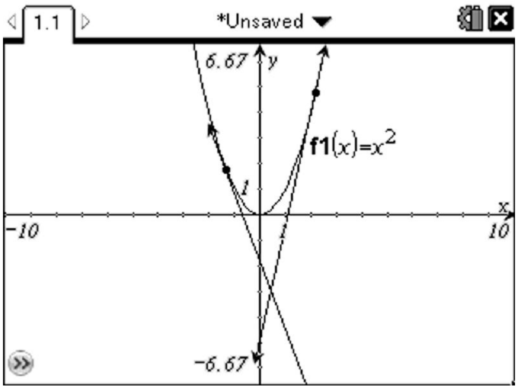
Construction Steps

1. Start a new document with a Graphs application.
2. Save it under the name 'Intersecting Tangents'
3. Type in x^2 and press ▪ to draw the graph of the function f(x)=x^2
4. Select the Tangent command and read how to use it:

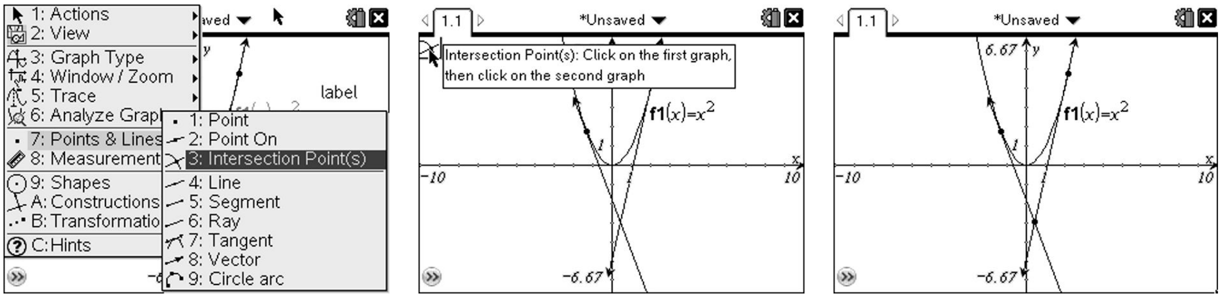




5. Construct two tangents on the parabola, and extend them downwards by grabbing the arrows on their ends:



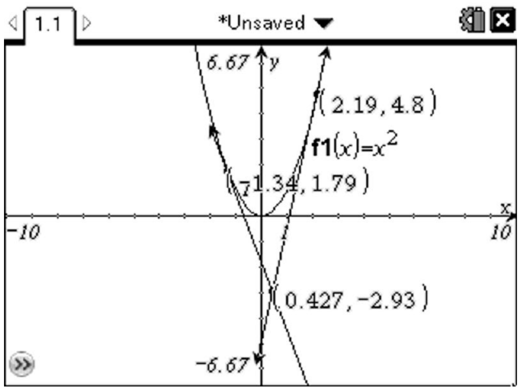
6. Add in the intersection point of the tangents, so that a new point appears.



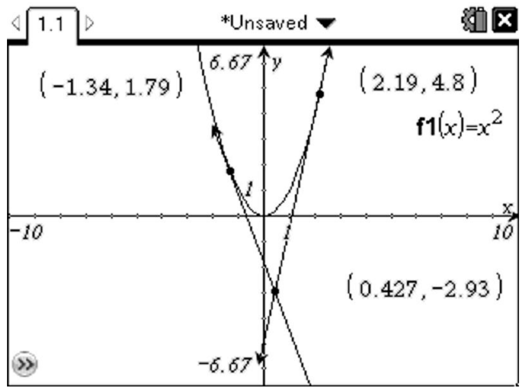
7. We now require the coordinates of <u>each</u> of the three points of interest.
Move over <u>each</u> point in turn, press / then b and select 'Coordinates and Equations' from the pop-up menu that appears.

8. Your display is probably looking quite untidy (as shown, below left) with slightly different coordinate values. Simply grab and move each of the text items to give a screen roughly like that shown, below right:

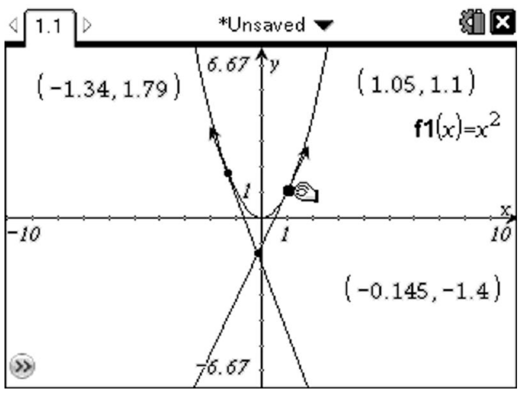
[it is important that you move each set of coordinates away from their point, else they will still be attached to the point, and they will move around the screen when the point moves.]



grab and move text around ⇒



9. Now grab <u>one</u> of the points on the parabola and move it along the parabola.
You will see the effect that it has on the intersection point.
Release the first point and grab the other point and move it around, if you want.



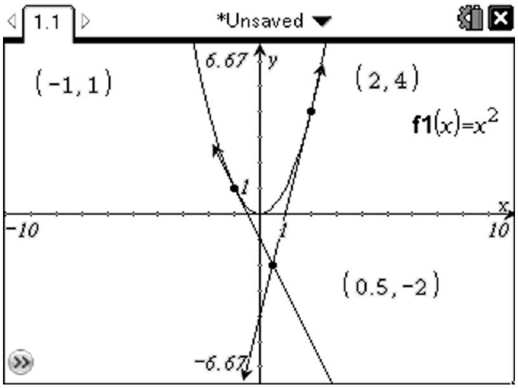
You might try and grab the intersection point, but you will not be able to.

Can you think <u>why</u> you <u>can't</u> grab and move the intersection point?

10. You've probably realised that all the decimal values for the coordinates make it hard to monitor the effect that moving the two points on the parabola are having on the location of the intersection point.

You can edit the x-coordinate of each of a point on the parabola, by moving over the x-coordinate, clicking on it **<u>twice</u>**, typing in a value for the x-coordinate and then pressing · .

Edit both points on the parabola to give this exact screen:



11. **You are now ready to start gathering information from your construction!**

The above screen shows that "the points (-1,1) and (2,4) give rise to an intersection point of (0.5, -2)"
You are recommended to edit the x-coordinates of the points on the parabola to explore their numerical effect on the coordinates of the intersection point.
Keep a written note of everything that you do, and any conclusions that you come to.
This stage may take some time and you might want to discuss your strategy with someone.

You should aim to reach a stage where if you knew the coordinates of two points on the parabola, say (a, a^2) and (b, b^2), then you could predict in terms of a and b the coordinates of the intersection point of the tangent lines.

12. You should be able to verify your finding with several numerical examples.
However, this does not **<u>prove</u>** your finding. *Can you think <u>why</u> it does <u>not</u> prove it?*

To prove your finding, you would most likely need to *algebraically* analyse the situation, rather than *numerically* analyse it.

<u>How to Algebraically Analyse the Intersection of two Tangent lines from a Parabola.</u>

i.) You need to find out the equation of each tangent line, in the form $y=mx+c$
For each line, you have the tangent point, say (a, a^2) and you need to know the gradient of the line.
More advanced mathematics tells us that for $f(x)=x^2$, the gradient at the point when $x=a$ is $2a$.
So, find the equation of the line that goes through (a, a^2) with gradient $2a$.

ii.) Then, find the equation of the line that goes through (b, b^2) with gradient $2b$.

iii.) Use your knowledge of solving simultaneous equations to find the intersection point of these two lines.

iv.) The result that you get should agree with your findings from step 11, above.

<u>Next Steps</u>
Mathematicians always ask more questions after they have solved a first 'situation'.
Here are some examples of what they might ask:

- What happens when instead of $f(x)=x^2$, it is $f(x)=2x^2$ or $f(x)=3x^2$?
- If I knew the coordinates of the intersection point, could I work out possible tangent points on the parabola? *[this is, in effect, the 'backwards' version of the process that you've already explored]*
- What happens when instead of $f(x)=x^2$, it is $f(x)=x^3$ or $f(x)=x^4$? *[this question leads to much more complex algebraic work. Increasing powers of x normally has this effect!]*

You should now try one of the first two questions – or your own – to see what you can find out.