



More Kilts and CAS

Session 217

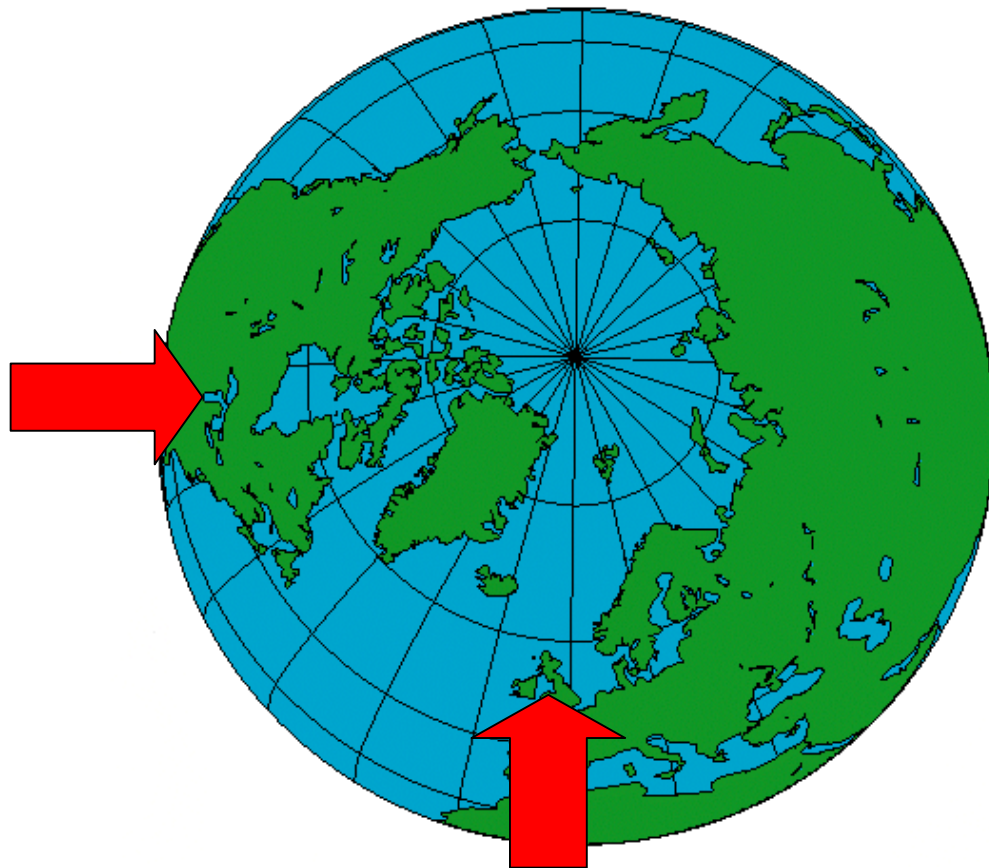
Saturday 3 March 2012

10:00 – 11:00am

**Nevil Hopley,
Edinburgh, Scotland**

T3 National Trainer

3710 Miles Away on a Bearing of 043°





**Strictly Limited Offer of Tartan
TI-Nspire CX Cases at the end of this talk.**

This talk will have a....

Beginning

Where am I coming from?

Other relevant background information.

Middle

Activities I have created and used.

Lessons learnt.

End

Student & Exam Preparation Issues

And you can download all that you see today from

www.CalculatorSoftware.co.uk/morecas

October 2008 – February 2011

A Brief History of that Time

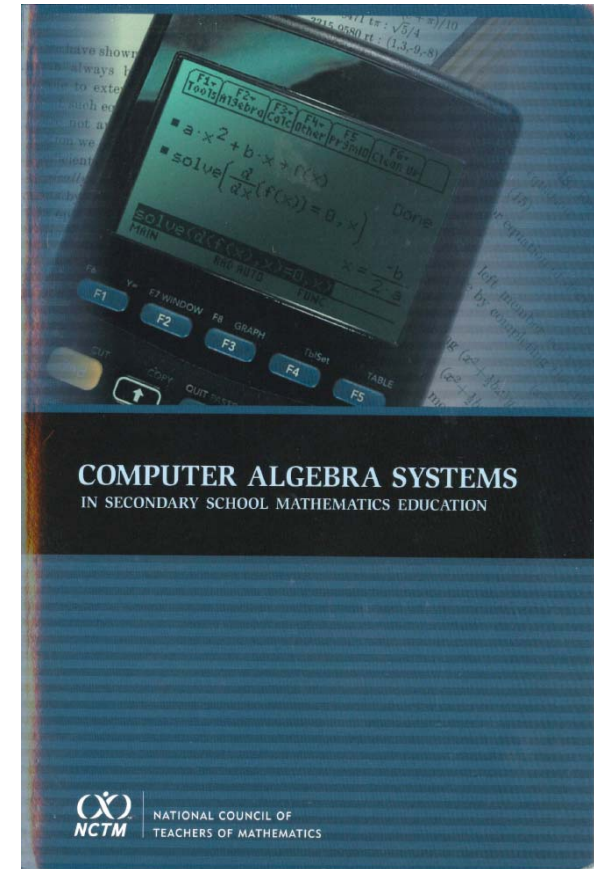
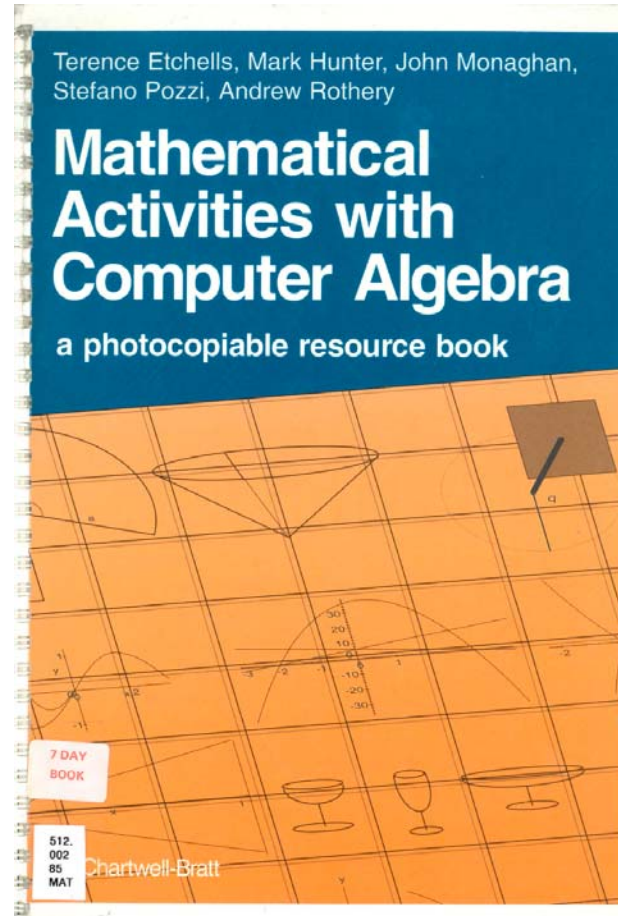
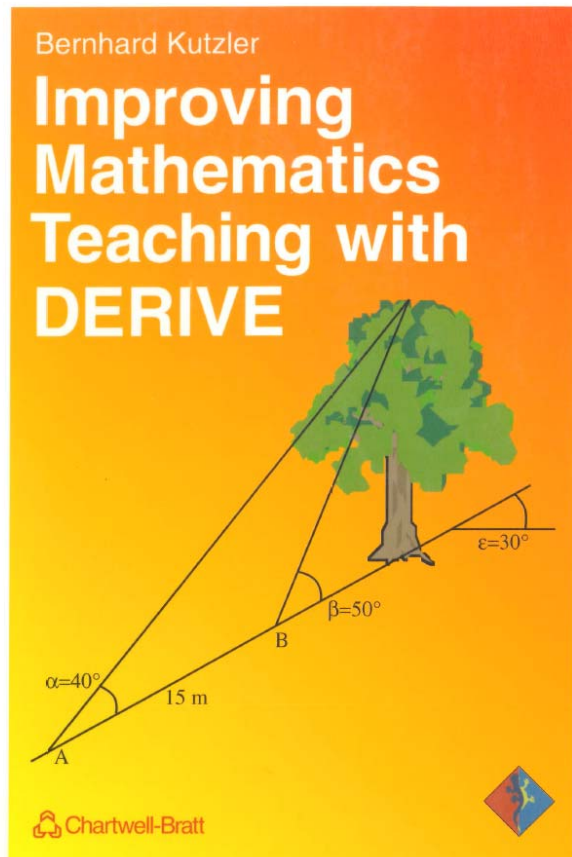
My T3 International San Antonio Talk

Talking to Conference Speakers

Classroom 'Research' aka 'Trying Things Out'

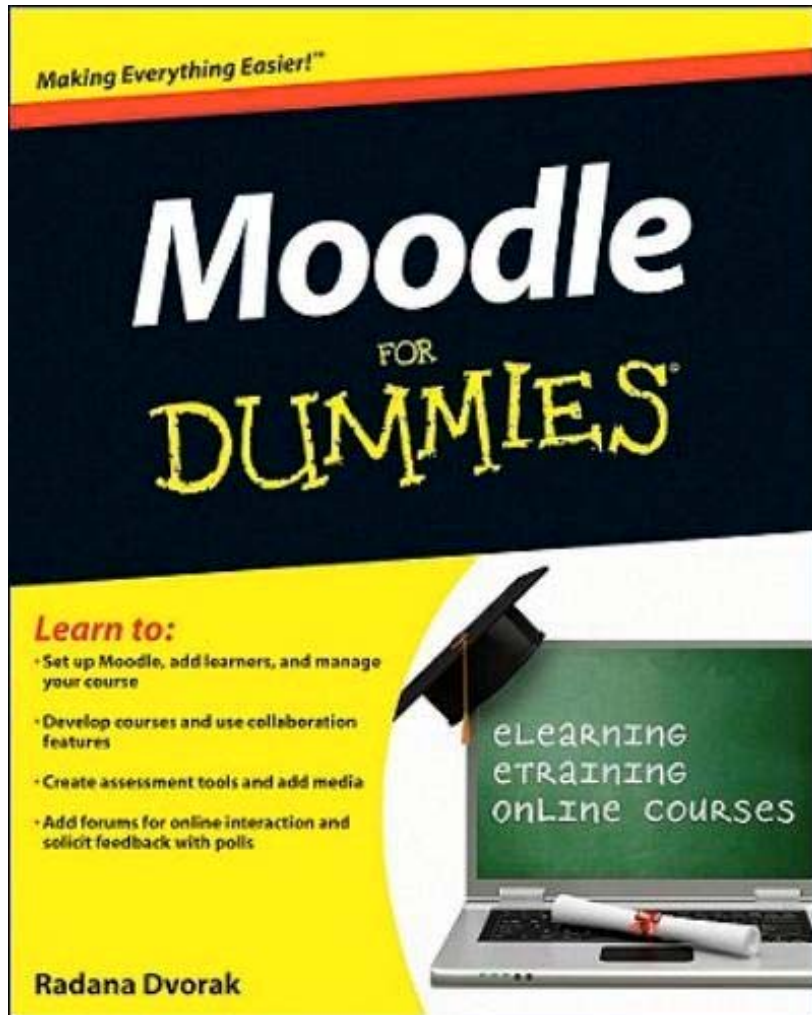
Books purchased...and read!

MyLib since February 2011

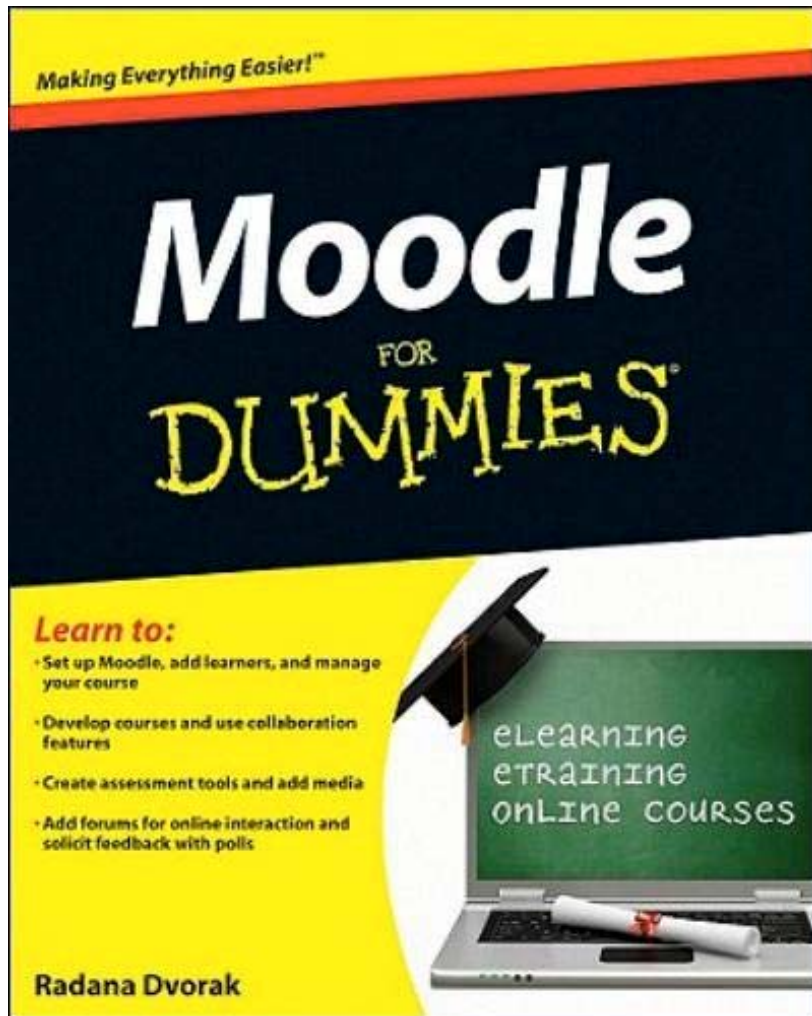


Generation Z (born 1995–2012)

... The learners of this generation are



Generation Z (born 1995–2012)



... The learners of this generation are impatient, seem to expect immediate results, and multitask with tech devices at exceptional speeds. They don't like to read instructions — most jump in and get on with it. Their expectations of technology are demanding. This generation will take to eLearning and will push boundaries.

Generation Z's Compatability with CAS

- ✓ Are play-oriented.
- ✓ Expect immediate results.
- ✓ Expect information to come to them or accessible at one click.
- ✓ Do not read instructions, especially step-by-step outlines, but jump straight in.
- ✓ Do not process as linearly as previous generations.
- ✓ Are impatient if technology is not quick enough — they find something else to do.
- ✓ Trust the medium.

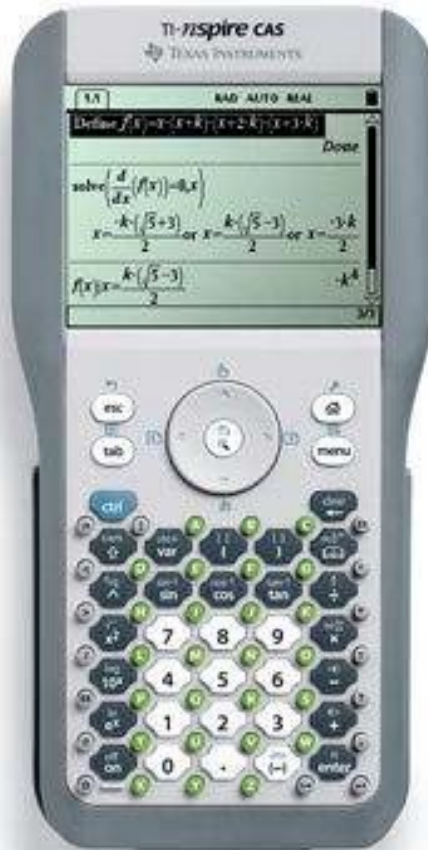
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- ✓ Do not read instructions, especially step-by-step outlines, but jump straight in.
- ✓ Do not process as linearly as previous generations.
- ✓ Are impatient if technology is not quick enough — they find something else to do.
- ✓ Trust the medium.
- ✗ Look at graphics first and access text-based media last.
- ✗ Process things at "twitch speed" (ie "more than 100 images a minute.")
- ✗ Do not stay with tasks as long.
- ✗ Do not expect things to go wrong.

Generation Z ... in Person!



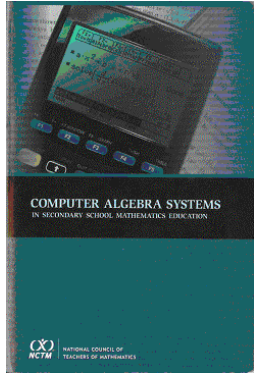
August 2011 to present



A class of 9 weaker students tackling topics such as:

- linear trigonometric equations
- solving simultaneous equations
- factorising trinomials
- length of sector arc
- sine & cosine rules
- basic rules of indices

Department Staff Training - September 2011



“Existing research suggests that the initial experience of teaching with CAS can be extremely challenging for teachers, and teachers’ instructional strategies unfold and change in unexpected ways during teachers’ early attempts in implementing CAS in their classrooms”

**“Using Research to Influence Teaching and Learning with CAS”
by Rose Mary Zbiek on page 199.**

Quote from one Teacher in my Department

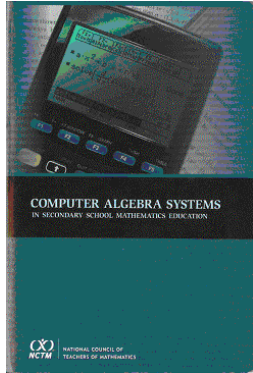
“Using CAS for ‘changing the subject’ made the pupils focus on the process and made me re-evaluate the way I teach solving equations, etc.

I usually write the process in brackets when first teaching equations and gradually remove this support as they become more familiar with the method.

Using CAS made me consider whether I have actually been removing the most important part of the working?”

Dr Nicky Armstrong

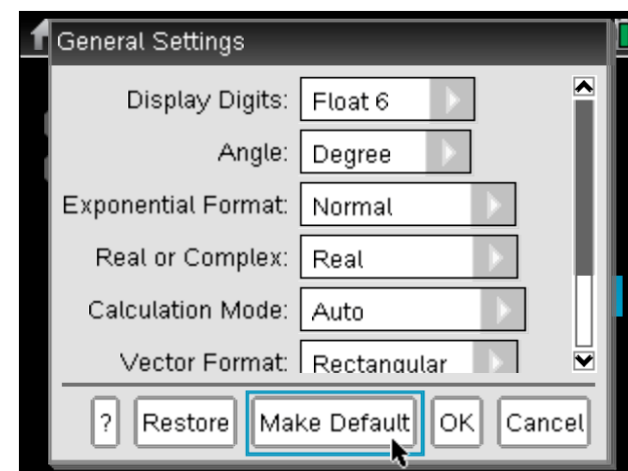
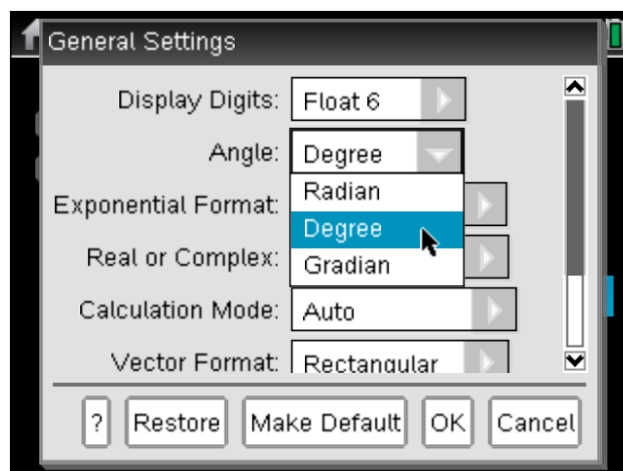
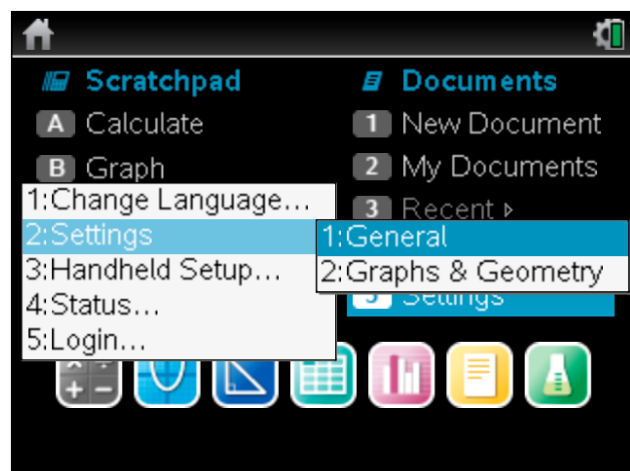
Department Staff Training - September 2011



“Time and resources for teacher preparation and planning are crucial, particularly given the high expectations for expertise in technology, mathematics and pedagogy”

**“Using Research to Influence Teaching and Learning with CAS”
by Rose Mary Zbiek on page 212.**

Settings > General > Degrees > Make Default



.....then open a Scratchpad calculator.

Issues with Elementary Trigonometry



and then try...



When does it 'switch' behaviour?

Solving Simple Linear Trigonometric Equations

Scratchpad

$$\sin(x)=0.4$$
$$\sin^{-1}(\sin(x)=0.4) \quad \sin^{-1}(\sin(x))=23.5782$$

|

2/99

Scratchpad

$$\tan(x)=0.4$$
$$\tan^{-1}(\tan(x)=0.4)$$
$$\text{mod}(x-90,180)-90=21.8014$$

|

⚠ Domain of the result might be larger than the ...

Solving Trig Equations CAS.tns

2.5 2.6 2.7 Solving Trig ...CAS

You will now have to use the other commands you've seen when you press the **VAR** button:

inversesine(*equation*)

inversecosine(*equation*)

inversetangent(*equation*)

2.9 2.10 2.11 *Solving Tri...CAS

© Press ENTER, then make the following read as x=....

$\tan(x)=0.247$	$\tan(x)=0.247$
$\text{inversetangent}(\tan(x)=0.247)$	$x=13.874$

3/99

Further ... Quicker

Feedback Messages

The screenshot shows a TI-84 Plus CE calculator screen with the title bar "*Solving Tri... CAS". The screen displays a sequence of steps for solving the equation $5 \sin(x) - 4 = 0$. The steps are as follows:

- © Press ENTER, then make the following read as $x = \dots$
- $5 \cdot \sin(x) - 4 = 0.$
- $(5 \cdot \sin(x) - 4 = 0.) + 4$
- $5 \cdot \sin(x) = 4.$
- $\text{inversesine}(5 \cdot \sin(x) = 4.)$
- "Inverse sine not appropriate"

The screen also shows a status bar at the bottom right indicating "4/99".

Solving Linear Equations CAS.tns

1.1 1.2 1.3 *Solving Lin... CAS

When you are done, you can also use the **check()** function.

Just press the **VAR** button to get the **check** command, and then paste the equation inside the brackets after the word **check**.

eg. **check**($x=7$) returns "Equation solved."

eg. **check**($x=2x+3$) returns "Not yet solved."

2.5 2.6 2.7 *Solving Lin... CAS

© Press ENTER, then make the following read as $x=...$

$3 \cdot x + 5 = 17$	$3 \cdot x + 5 = 17$
$(3 \cdot x + 5 = 17) - 5$	$3 \cdot x = 12$
$\frac{3 \cdot x = 12}{3}$	$x = 4$
$check(x=4)$	"Equation solved for x."

5/99

Feedback Messages

2.6 2.7 2.8 *Solving Lin... CAS

read as x=....

$7 + 4 \cdot x = 10 \cdot x - 13$	$4 \cdot x + 7 = 10 \cdot x - 13$
$(4 \cdot x + 7 = 10 \cdot x - 13) - 7$	$4 \cdot x = 10 \cdot x - 20$
$\frac{4 \cdot x = 10 \cdot x - 20}{4}$	$x = \frac{5 \cdot (x - 2)}{2}$
$check\left(x = \frac{5 \cdot (x - 2)}{2}\right)$	"Not yet solved."

5/99

Rearranging the Cosine Rule (aka 'Law of Cosines')

Matthew



Jon



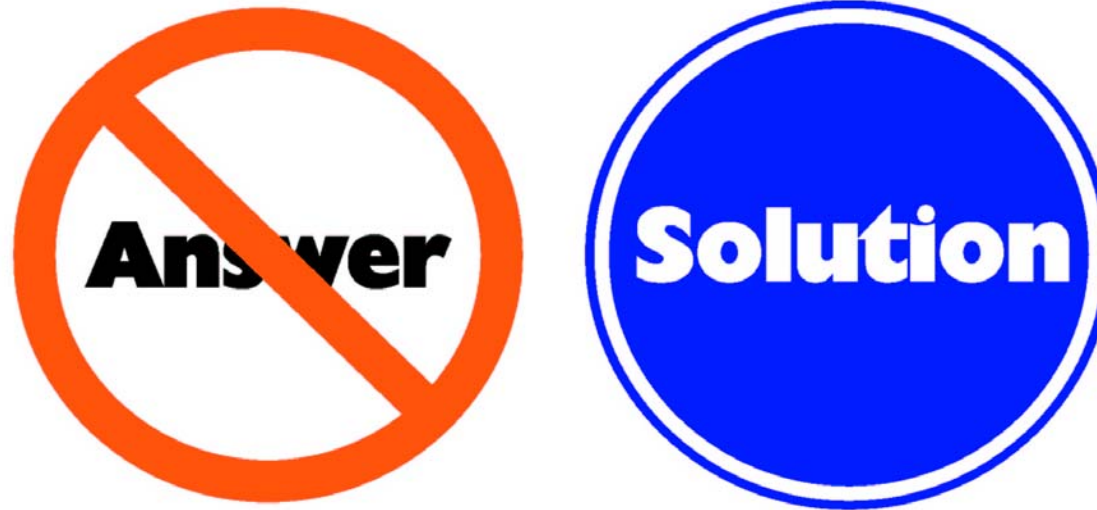
Joanna



Elizabeth



Follow Up to Rearranging the Cosine Rule



They did it by CAS.

They did it on paper.

I did it on the board.

Play spot the difference!

Cosine Rule Rearranging Discussion

$$\cos(\text{angl}.a) = \frac{a^2 - b^2 - c^2}{-2bc} \quad \cos(\text{angl}.a) = \frac{-(a^2 - b^2 - c^2)}{2bc} \quad \cos(\text{angl}.a) = \frac{b^2 + c^2 - a^2}{2bc}$$

Dealing with that negative sign....

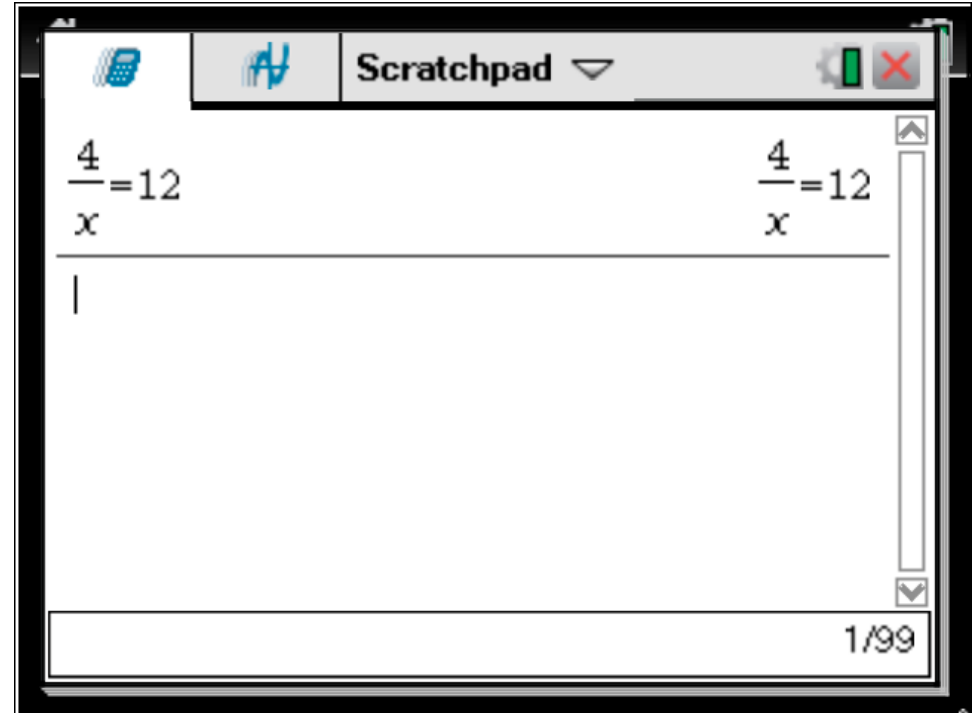
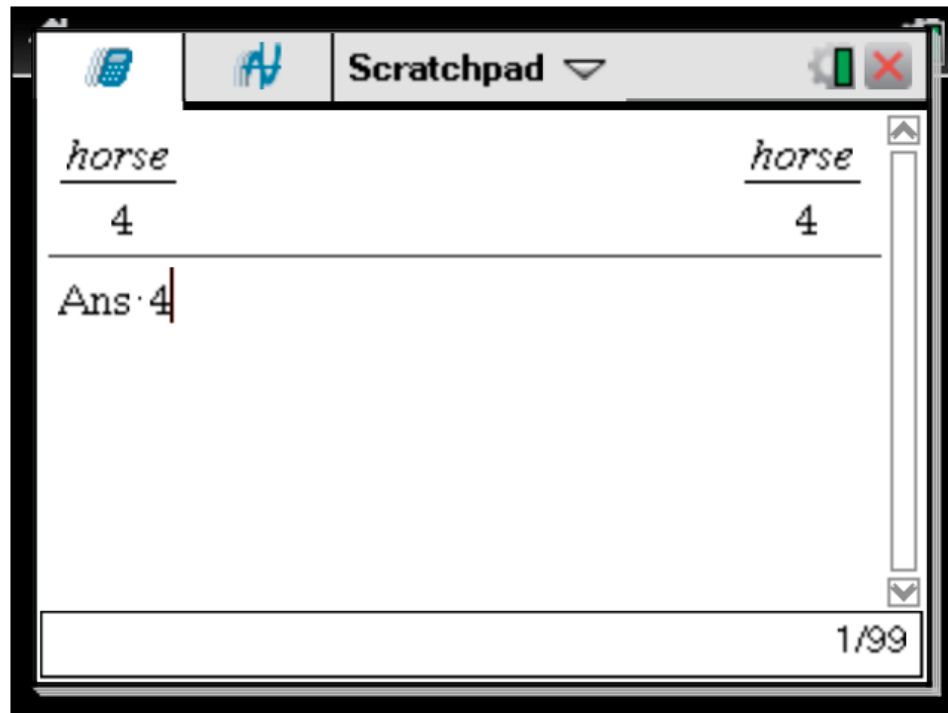
10 minute video clip of this discussion on



url link at

www.calculatorsoftware.co.uk/morecas

Preparation for Rearranging the Sine Rule (aka 'Law of Sines')



'The Lesson that Crashed'

Algebraic & Trigonometric Identities

Scratchpad

$4 \cdot x + 7 = 2 \cdot x - 6$	$4 \cdot x + 7 = 2 \cdot x - 6$
$4 \cdot x + 7 = 2 \cdot x - 6 x = 8$	false
$4 \cdot x + 7 = 2 \cdot x - 6 x = -6.5$	true

3/99

Scratchpad

$x + x = 2 \cdot x$	true
$(a+b)^2 - (a-b)^2 = 4 \cdot a \cdot b$	true
$\frac{\sin(x)}{\cos(x)}$	$\tan(x)$
$(\sin(x))^2 + (\cos(x))^2$	1

4/99

'Proving It' 1 of 3

1.1	*cas5 ▾	⚙️ 🔴
$\frac{\sin(x)}{\cos(x)} = \tan(x)$	true	⬆️
$(\sin(x))^2 + (\cos(x))^2$	1	
$\frac{\sin(x)}{\cos(x)} = \tan(x) x=7$	true	
$(\sin(x))^2 + (\cos(x))^2 = 1 x=0$	true	
		⬇️
8/99		

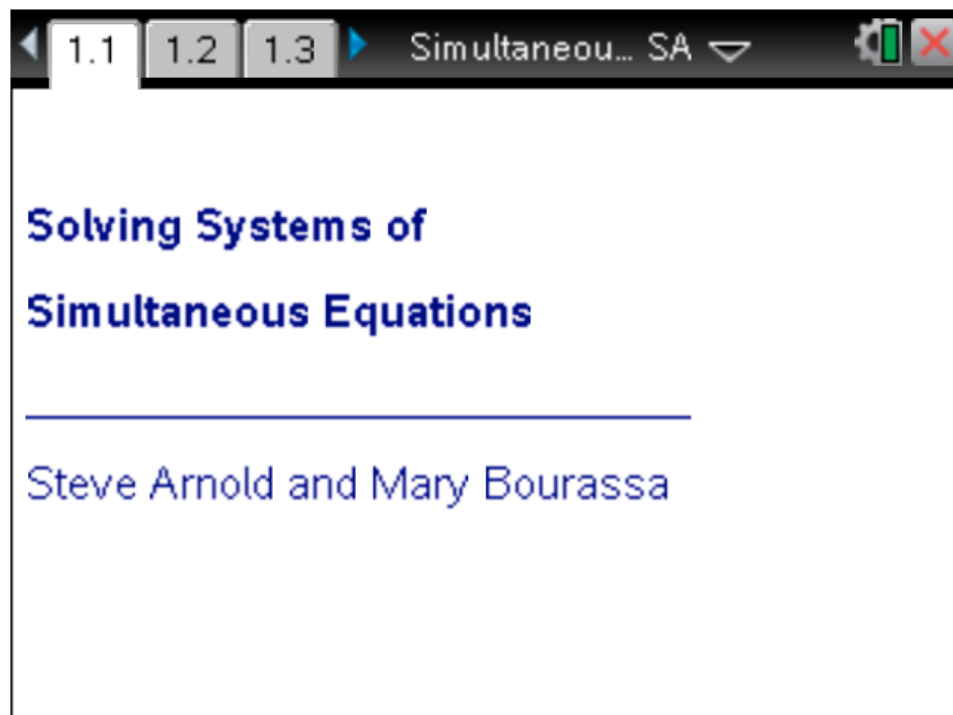
'Proving It' 2 of 3

$x+x=2 \cdot x$	true
$(a+b)^2 - (a-b)^2 = 4 \cdot a \cdot b$	true
$\frac{\sin(x)}{\cos(x)}$	$\tan(x)$
$\frac{\sin(x)}{\cos(x)} = \tan(x)$	true
$(\sin(x))^2 + (\cos(x))^2$	1
$(\sin(x))^2 + (\cos(x))^2 = 1$	true
$\sin(x) = \cos(x) \cdot \tan(x)$	true
$(\sin(x))^2 + (\tan(x))^2$	$(\tan(x))^2 \cdot ((\cos(x))^2 + 1)$
$\cos(x) + \tan(x)$	$\frac{(\cos(x))^2 + \sin(x)}{\cos(x)}$
$\sin(3) = \cos(3) \cdot \tan(3)$	true
13/99	

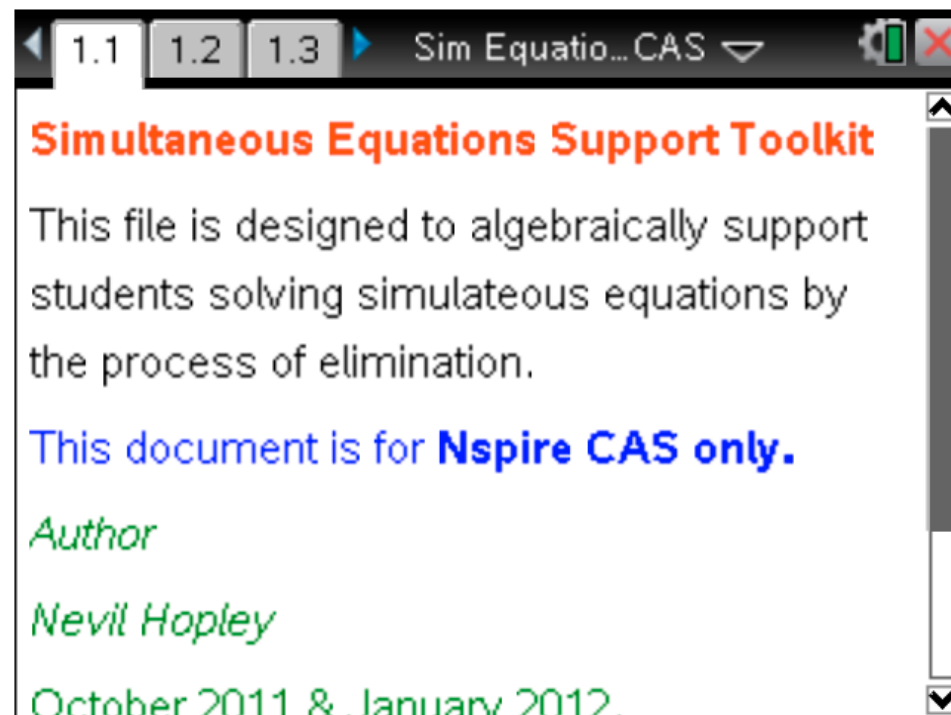
'Proving It' 3 of 3

$x+x=2 \cdot x$	true
$\frac{\sin(x)}{\cos(x)}$	$\tan(x)$
$\cos(x) \cdot \tan(x)$	$\sin(x)$
$(\sin(x))^2 + (\cos(x))^2$	1
$(\sin(x))^2 + (\cos(x))^2 = (\tan(x))^2$	$1 = (\tan(x))^2$
$(\tan(x))^2$	$(\tan(x))^2$
$(\tan(x))^2 = (\sin(x))^2 + (\cos(x))^2$	$(\tan(x))^2 = 1$
$(\tan(x))^2 = 1$	$(\tan(x))^2 = 1$
$\text{solve}((\tan(x))^2 = (\sin(x))^2 + (\cos(x))^2, x)$	$x = n1 \cdot \pi + \frac{\pi}{4} \text{ or } x = n2 \cdot \pi - \frac{\pi}{4}$
$\text{solve}((\tan(x))^2 = (\sin(x))^2 + (\cos(x))^2, x)$	$x = n3 \cdot \pi + \frac{\pi}{4} \text{ or } x = n4 \cdot \pi - \frac{\pi}{4}$
13/99	

Simultaneous Linear Equations – a choice!



Simultaneous_Eqns SA.tns
(for Nspire non-CAS)



Sim Equations Toolkit CAS.tns
(for Nspire CAS)

Sim Equations Toolkit CAS.tns

The screenshot shows a software window titled '*Sim Equati... CAS'. It contains a table with two columns of equations. The first column contains the original equations and their manipulation steps using standard arithmetic notation. The second column shows the resulting simplified equations. The equations are:

$x+2\cdot y=12$	$x+2\cdot y=12$
$3\cdot x-4\cdot y=15$	$3\cdot x-4\cdot y=15$
$(x+2\cdot y=12)\cdot 2$	$2\cdot (x+2\cdot y)=24$
$(3\cdot x-4\cdot y=15)+(2\cdot (x+2\cdot y)=24)$	$5\cdot x=39$

The bottom right corner of the window displays '4/99'.

Without Toolkit

...excessive brackets!

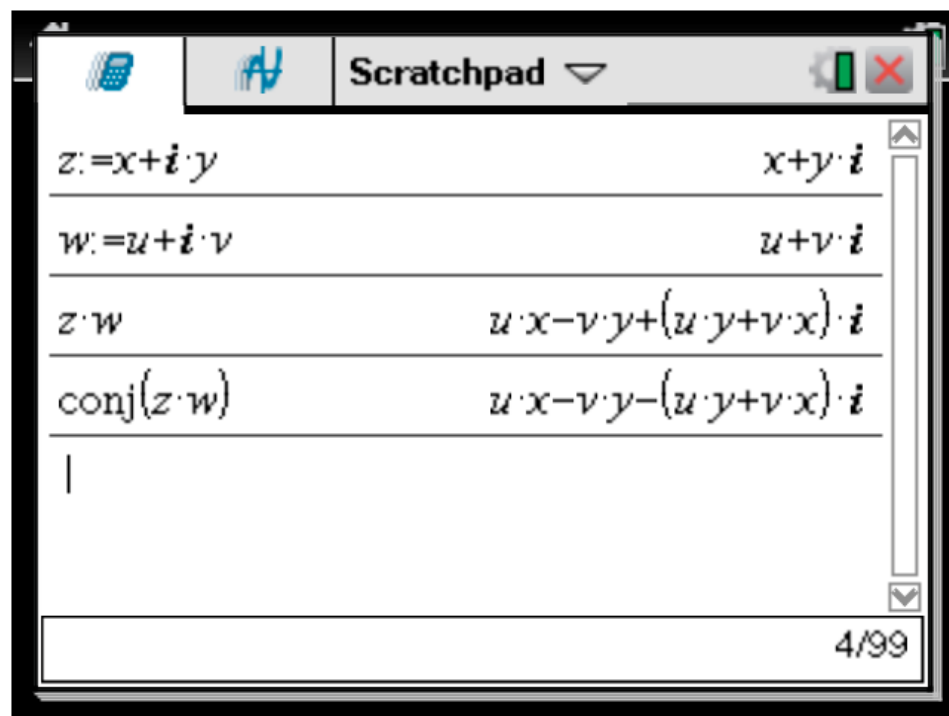
The screenshot shows the same software window as the previous one, but with toolkit commands used for the manipulation steps. The equations are:

$x+2\cdot y=12$	$x+2\cdot y=12$
$3\cdot x-4\cdot y=15$	$3\cdot x-4\cdot y=15$
$multiplyby2(x+2\cdot y=12)$	$2\cdot x+4\cdot y=24$
$add(3\cdot x-4\cdot y=15, 2\cdot x+4\cdot y=24)$	$5\cdot x=39$

The bottom right corner of the window displays '4/99'.

Using Toolkit Commands

Conjugates of Complex Numbers

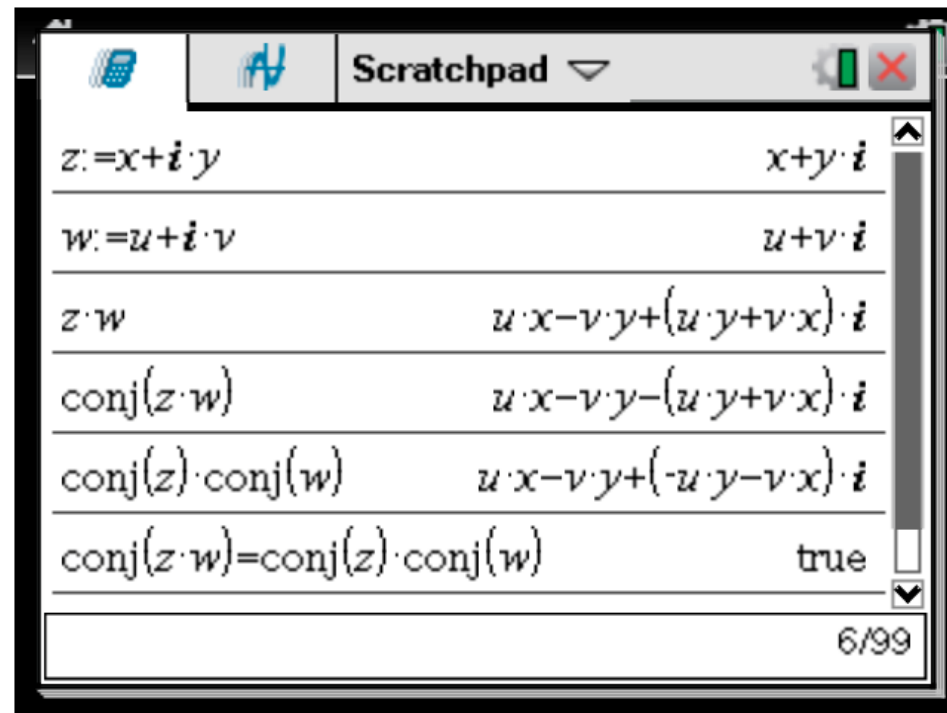


A screenshot of a "Scratchpad" window, likely from a presentation software. The window has a title bar with a calculator icon, a text icon, and the text "Scratchpad" followed by a dropdown arrow. On the right side of the title bar are window control icons (minimize, maximize, close). The main area of the window contains a table of complex number operations and their results, separated by horizontal lines. The entries are:

$z := x + i \cdot y$	$x + y \cdot i$
$w := u + i \cdot v$	$u + v \cdot i$
$z \cdot w$	$u \cdot x - v \cdot y + (u \cdot y + v \cdot x) \cdot i$
$\text{conj}(z \cdot w)$	$u \cdot x - v \cdot y - (u \cdot y + v \cdot x) \cdot i$

At the bottom right of the window, there is a status bar showing "4/99".

Conjugate of Product = Product of Conjugates?



A screenshot of a 'Scratchpad' window with a title bar containing a calculator icon, a pencil icon, and the text 'Scratchpad'. The window contains a table of algebraic expressions and their simplified forms. The table has six rows. The first two rows define complex numbers z and w in terms of real and imaginary parts. The next three rows calculate the product $z \cdot w$, its conjugate, and the product of the individual conjugates. The final row shows that the conjugate of the product equals the product of the conjugates, with the result 'true'. A status bar at the bottom right shows '6/99'.

$z := x + i \cdot y$	$x + y \cdot i$
$w := u + i \cdot v$	$u + v \cdot i$
$z \cdot w$	$u \cdot x - v \cdot y + (u \cdot y + v \cdot x) \cdot i$
$\text{conj}(z \cdot w)$	$u \cdot x - v \cdot y - (u \cdot y + v \cdot x) \cdot i$
$\text{conj}(z) \cdot \text{conj}(w)$	$u \cdot x - v \cdot y + (-u \cdot y - v \cdot x) \cdot i$
$\text{conj}(z \cdot w) = \text{conj}(z) \cdot \text{conj}(w)$	true

6/99

“That’s a bit of a cheat”

“It’s just testing a statement and not getting you thinking”

Their Conjectures

$$-\text{conj}(z) = \text{conj}(-z)$$

$$\text{conj}(z+w) = \text{conj}(z) + \text{conj}(w)$$

$$\text{conj}(z/w) = \text{conj}(z) / \text{conj}(w)$$

$$\text{conj}(z-w) = \text{conj}(z) - \text{conj}(w)$$

They said “Let’s dabble in powers” !

$$\text{conj}(z^2) = (\text{conj}(z))^2$$

$$\text{conj}(\sqrt[3]{z}) = \sqrt[3]{\text{conj}(z)}$$

$$\text{conj}(z^w) = (\text{conj}(z))^w \text{ or } z^{\text{conj}(w)} ?$$

$$\text{conj}(\text{conj}(z) \cdot z) = z \cdot \text{conj}(z)$$

Off Limits to Manual Calculation.

Conceptually Stimulating!

Invariant Logarithmic Chickens!

$\frac{\log_a(40)}{\log_a(5)}$	$\frac{\log_a(40)}{\log_a(5)}$
$\frac{\log_a(40)}{\log_a(5)}$	2.29203
$\frac{\log_a(40)}{\log_a(5)} _{a=8}$	2.29203
$\frac{\log_a(40)}{\log_a(5)} _{a=chicken}$	2.29203
4/99	

Levels of Student Awareness

Seeking a Solution

Needing Guidance

Objective

Lines of Working

Course Assessment

Solution Aware

“you can concentrate on how to answer the questions because it does the algebra for you so you don't have to think about the algebra as well”

Thomas

“The CAS really helps with difficult equations that I can't tackle on my own. Seeing the answer before you know how to get there can really help to get the solution.”

Ghazi

Guidance Aware

"I have used the system to check the answer I have got when I have been unsure and also when I have no clue what I am supposed to be doing"

Alan

"pupils can turn to it, for another option if they are stuck on a problem, for guidance"

Ben

"it can tell you whether what you are inclined to do is right or wrong"

Hannah

Objective Aware

“can give you the end goal of a task”

Elizabeth K.

“help to quickly do complex algebra – to show things being taught in a new light”

David W.

Working Aware

“We can check equations, both the final answer and individual lines of working”

Catriona S.

“being able to check the stages in your work rather than just the answer”

Lucy

Assessment Aware

“Because we use the calculators so much in class I feel like I rely on it too much to get the answers, and would be lost in an exam situation without it.”

Rosanna

“I also didn't know how to factorise without using the CAS handheld so in an exam it wouldn't have helped me.”

Anon

Preparing for Exams

**CAS can give you the answers,
so it's the process that becomes important**

Implications for Nspire non-CAS usage?

- I set past Exam Questions for homework.
- I provide them with videos of Nspire solutions ahead of due date.
- Will it switch emphasis from Answer to Solution?

Final Comment

“Sometimes I feel it adds an extra dimension which mostly helps you to understand it better.

However sometimes it can hinder your learning by overcomplicating things.”

Jon



Only 8 in the whole world!

Suggested minimum
donation of \$15.....

All proceeds to



Want More of Kilts & CAS?

www.CalculatorSoftware.co.uk/morecas

Want More of Me?

CAS Power Session

"International Perspectives on Using CAS Technology"

Tomorrow Morning, 8:30-10:30am

Tell your friends!

Thank you for coming to my talk.

Nevil Hopley

T3 National Trainer,
Scotland & UK.