



Kilts and CAS

Session 131

Friday 25 February 2011

1:15 – 2:15pm

**Nevil Hopley,
Edinburgh, Scotland**

T3 National Trainer



**Strictly Limited Offer of Royal Stewart Tartan
TI-Nspire Cases at the end of this talk.**

This talk will have a....

Beginning

What was the motivation?

What were the hurdles?

Middle

Activities I have created and used.

Other stuff to watch out for when using CAS.

End

Where to next?

And you can download all that you see today from

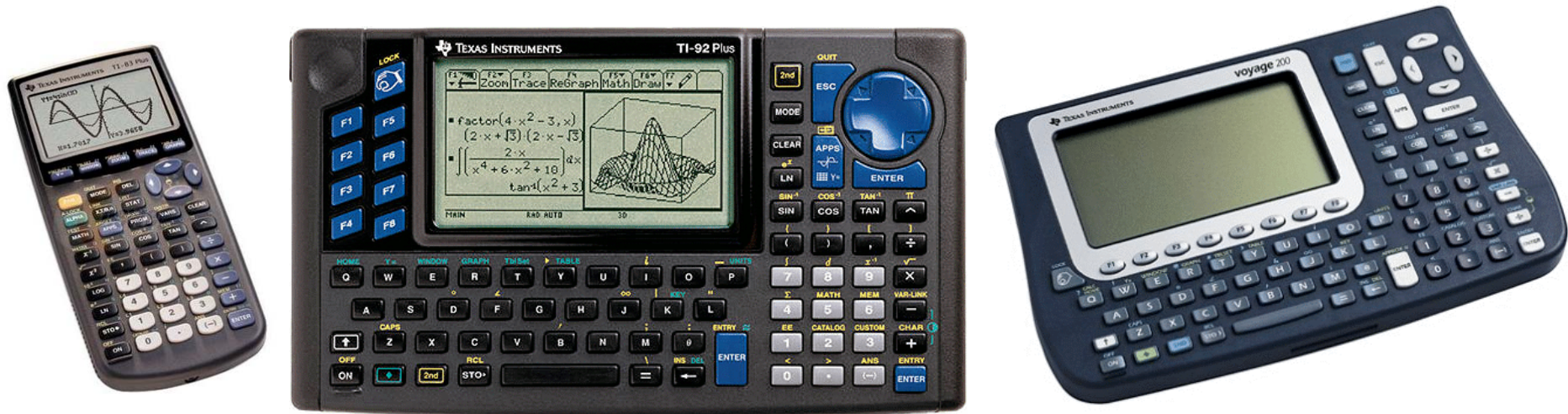
www.CalculatorSoftware.co.uk/cas

A Gamble? An Uphill Struggle?

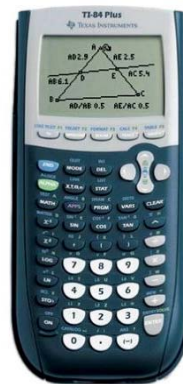


Why now?

Previously...



Not the same as...





Scottish Qualification Authority Module

“Modelling with Mathematics”

linking CAS and Dynamic
Geometry

.....shelved!

Two Flavours of Same Handheld



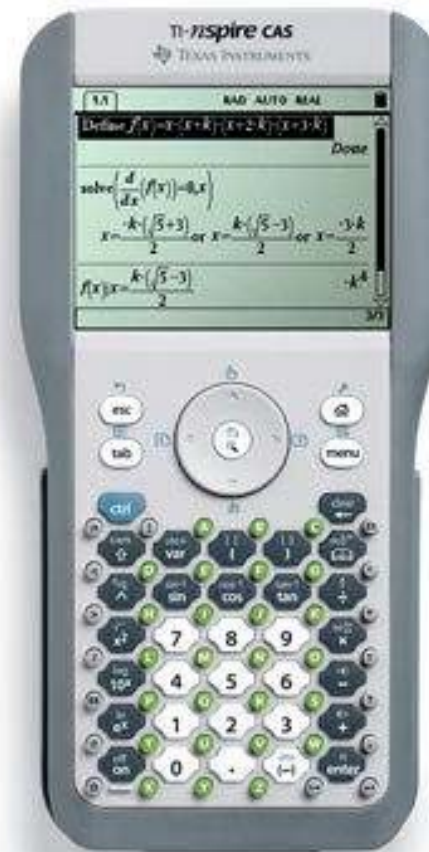
No Compatibility or Use-ability Hurdles to Overcome

What first?

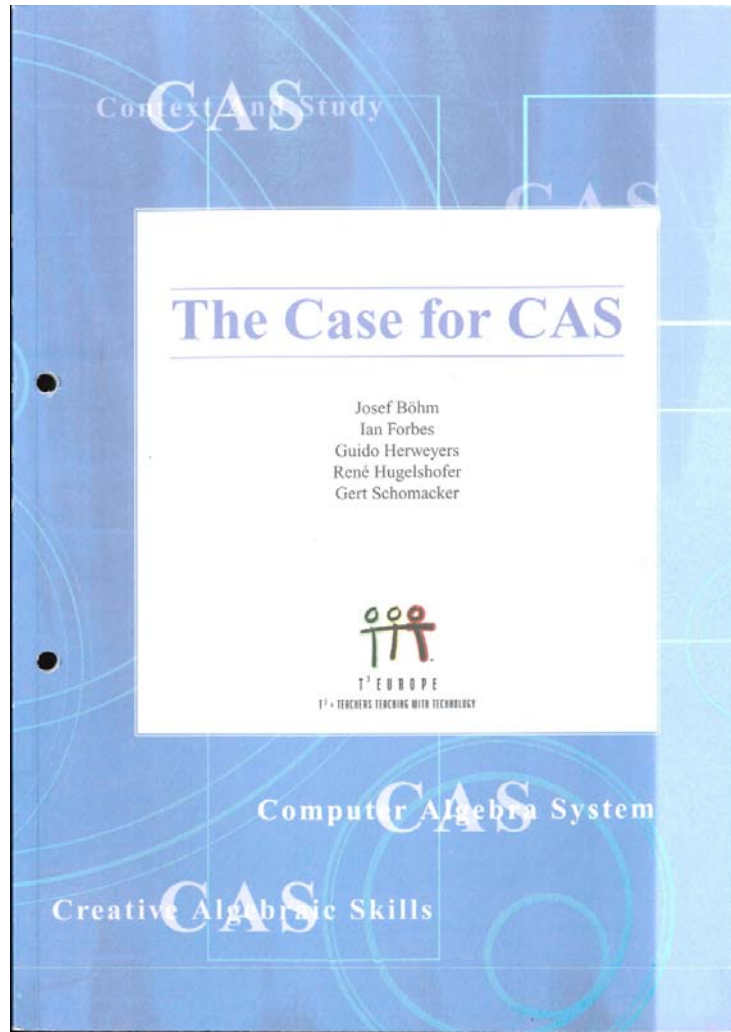


Nspire Navigator Pilot Site, October 2008

32x



What was out there?



The Case for CAS

www.t3ww.org/cas

Mainly calculus and optimisation activities.



**2009 T3 International Conference
Seattle, WA**

**2010 T3 International Conference
Atlanta, GA**

Lynn Adsit
Russell Boyle
Jim Breunlin
Peter Fox
Jessica Kachur
Kim Schjelderup

Then I was off ! but to do what?

Nspire **CAS**, not just Nspire.

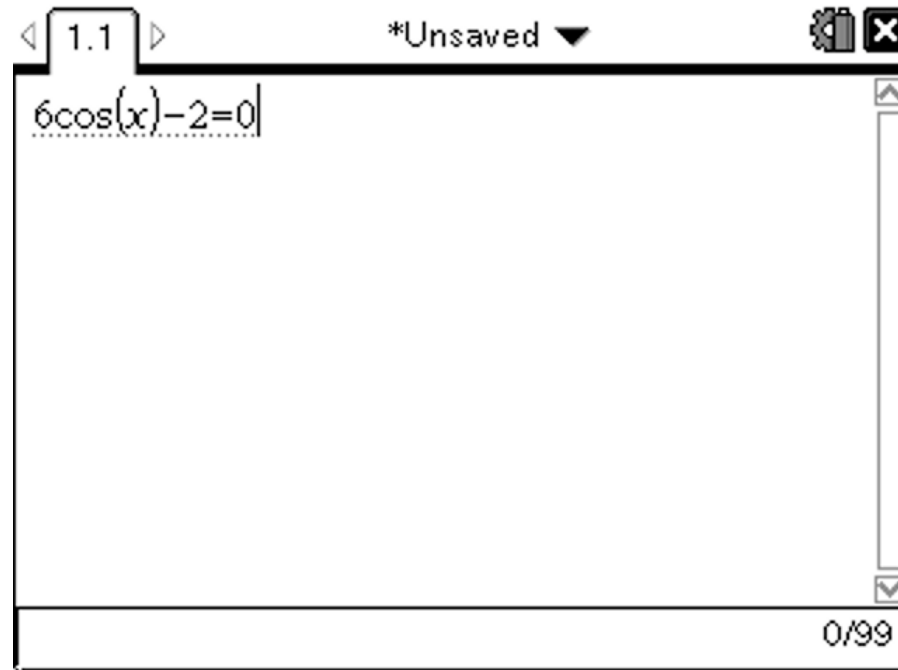
Algebraic Fractions with class of 13 year olds

The image shows three screenshots of a TI-Nspire CAS interface, arranged horizontally. Each window has a title bar with navigation buttons and a 'Fractions CAS' menu.

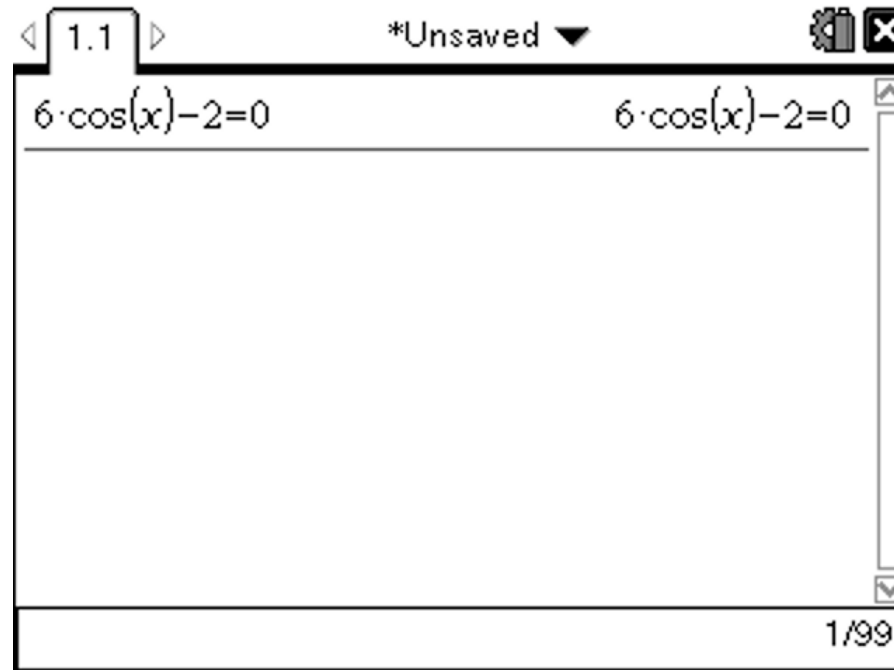
- Left window (Page 1.4):** The prompt is "© Now type in : $\frac{x}{3} + \frac{4}{5}$ ". The input field is empty. The page number "1/99" is at the bottom.
- Middle window (Page 1.4):** The text reads: "© That last page didn't do much. We need to tell the Nspire to give us the answer as a single fraction. To do this, press MENU > Number > Fraction Tools > Common Denominator so that you can type in the following: $\text{comDenom}\left(\frac{x}{3} + \frac{4}{5}\right)$ ". The input field is empty. The page number "1/99" is at the bottom.
- Right window (Page 3.6):** The prompt is "© Now type in: $\text{expand}\left(\frac{2a-3}{a+6}\right)$ ". Below it, the text says "© Construct the workings to support this answer." The input field is empty. The page number "2/99" is at the bottom.

My first attempt!

Solving Trigonometric Equations - 1

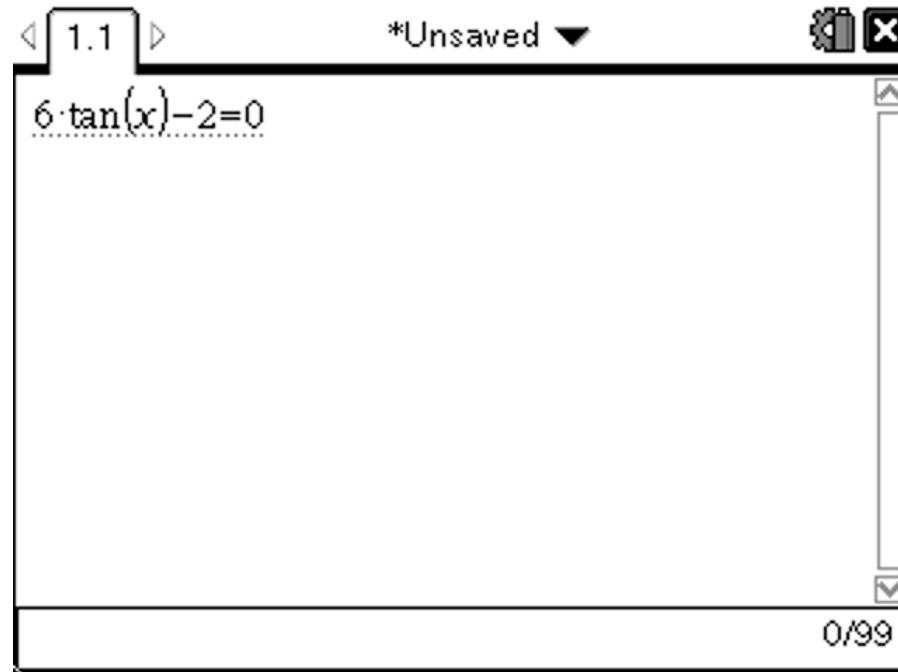


Solving Trigonometric Equations - 2



Inclusion of dot for multiplication really emphasised the processes going on.

Unexpected Things - 1



Unexpected Things - 2

The screenshot shows a software window with a title bar containing a tab labeled "1.1", the text "*Unsaved", and window control icons. The main content area displays two equations side-by-side, separated by a horizontal line. The left equation is $6 \cdot \tan(x) - 2 = 0$. The right equation is $\frac{-2 \cdot (\cos(x) - 3 \cdot \sin(x))}{\cos(x)} = 0$. A vertical scrollbar is visible on the right side of the content area. At the bottom right corner of the window, the page number "1/99" is displayed.

Unexpected Things – 3

The screenshot shows a software window titled "1.1" and "*Unsaved". It displays the following mathematical steps:

$$6 \cdot \tan(x) - 2 = 0 \qquad \frac{-2 \cdot (\cos(x) - 3 \cdot \sin(x))}{\cos(x)} = 0$$

$$\left(\frac{-2 \cdot (\cos(x) - 3 \cdot \sin(x))}{\cos(x)} = 0 \right) + 2 \qquad 6 \cdot \tan(x) = 2$$

The bottom right corner of the window shows "2/99".

Experiment as much as you can beforehand!

Rearranging Equations

Nspire CAS's auto-simplification of expressions

What do we do when we think about rearranging formulae?

I now know why students find rearranging so tricky!

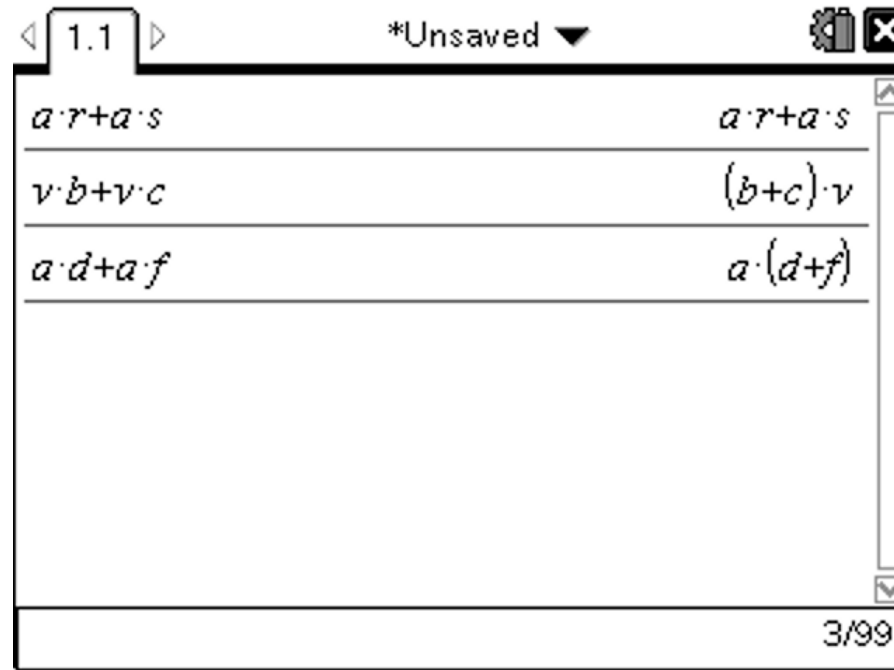
Unexpected Things - 4

1.1 *Unsaved

$a \cdot r + a \cdot s$	$a \cdot r + a \cdot s$
$v \cdot b + v \cdot c$	$(b+c) \cdot v$

2/99

Unexpected Things – 5



a q are constants
r z are variables

...with thanks to Peter Flynn, Melbourne, Australia.

Unexpected Things – 6

The screenshot shows a software window with a title bar containing a tab labeled "1.1", the text "*Unsaved", and window control icons. The main content area is a text editor with a white background and a black border. It contains the following text:

$a-x=b$	$a-x=b$
$2 \cdot (a-x)=b$	$-2 \cdot (x-a)=b$

The text is arranged in two columns, separated by a vertical line. The first row shows $a-x=b$ on both sides. The second row shows $2 \cdot (a-x)=b$ on the left and $-2 \cdot (x-a)=b$ on the right. The third row shows a vertical bar on the left. A vertical scrollbar is on the right side of the text area. At the bottom right corner of the window, the text "2/99" is displayed.



Unexpected Things - 7

The image shows a screenshot of a document editor window. The window title bar includes a tab labeled "1.1", the text "*Unsaved" with a dropdown arrow, and standard window control icons (minimize, maximize, close). The main content area contains two rows of mathematical expressions, each separated by a horizontal line. The first row shows $\frac{2-x}{7}$ on the left and $\frac{-(x-2)}{7}$ on the right. The second row shows $\frac{2-x}{-7}$ on the left and $\frac{x-2}{7}$ on the right. A vertical scrollbar is visible on the right side of the content area. At the bottom right corner of the window, the page number "2/99" is displayed.

$\frac{2-x}{7}$	$\frac{-(x-2)}{7}$
$\frac{2-x}{-7}$	$\frac{x-2}{7}$



2/99

Part 1 – Preparation for CAS - 1

3.6 4.1 4.2 ▶ Part 1 Rearr...lae ▼  

$p+h$ is the same as

- $h+p$
- $-p-h$
- $-(-p-h)$
- $-(h-p)$
- $p-(-h)$

4.3 4.4 4.5 ▶ Part 1 Rearr...lae ▼  

$a \cdot k + a \cdot p$ is the same as

- $a \cdot (p+k)$
- $a \cdot (k+p)$
- $(p+k) \cdot a$
- $(k+p) \cdot a$
- $a \cdot p + a \cdot k$
- $k \cdot a + p \cdot a$

Part 1 – Preparation for CAS – 2

4.1 4.2 4.3 Part 1 Rearr...lae

Dividing x by 2 is the same as

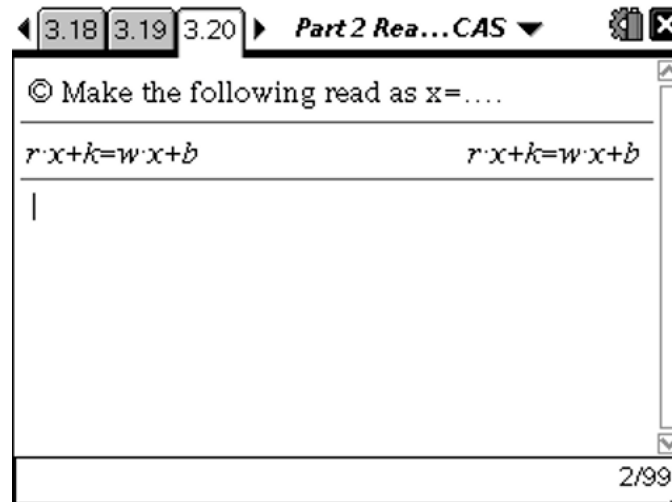
- $\frac{1}{2}x$
- $\frac{x}{2}$
- $x \div 2$
- $2x$
- $x/2$
- $0.5x$
- $\frac{2}{x}$
- \sqrt{x}

4.6 4.7 4.8 Part 1 Rearr...lae

$4a+12$ divided by 2 is

- $\frac{4a}{2}+12$
- $\frac{4a}{2} + \frac{12}{2}$
- $4a + \frac{12}{2}$
- $\frac{1}{2}(4a+12)$
- $(4a+12) \div 2$
- $4a+12 \div 2$

Part 2 – Rearranging Formulae Using CAS



 Video of Kai solving this.

www.CalculatorSoftware.co.uk/cas

Kai's Jotter

$$3.1.7) \quad rx + k = wx + b$$

$$rx = wx + b - k$$

$$rx - wx = b - k$$

$$x(r - w) = b - k$$

$$x = \frac{b - k}{r - w}$$

Kai's Test Homework (2 weeks later)

$$14) R = \frac{rs}{r+s}$$

$$R(r+s) = rs$$

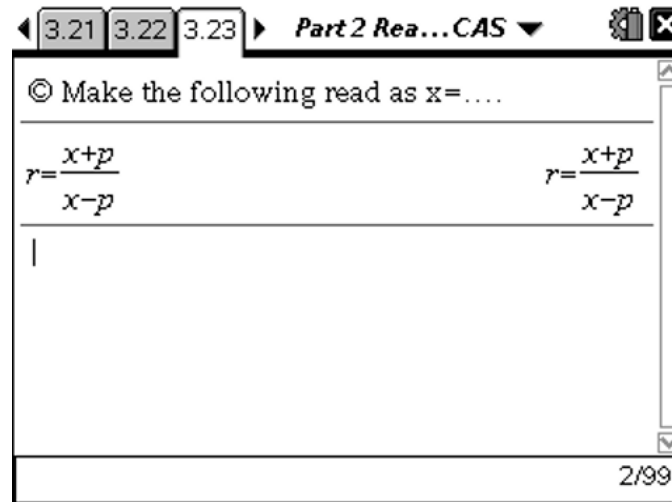
$$Rr + Rs = rs$$

$$Rr = rs - Rs$$

$$Rr = s(r - R)$$

$$\frac{Rr}{r - R} = s$$





 Video of Thomas solving this.

www.CalculatorSoftware.co.uk/cas

Thomas's Jotter

$$(22) \quad r = \frac{x+p}{x-p}$$

$$r(x-p) = x+p$$

$$x-p = \frac{x+p}{r}$$

$$x = \frac{x+p}{r} + p$$

$$\frac{x-x}{r} = \frac{p}{r} + p$$

$$xr - x = p + pr$$

$$x(r-1) = p(r+1) \quad x = \frac{p(r+1)}{r-1}$$

Thomas's Test Homework (2 weeks later)

(14.)

$$r = \frac{rs}{r+s}$$

$$r^2s = rs$$

$$rs = \frac{\cancel{rs}}{\cancel{r}}$$

$$rs = s$$

$$rs - s = 0$$

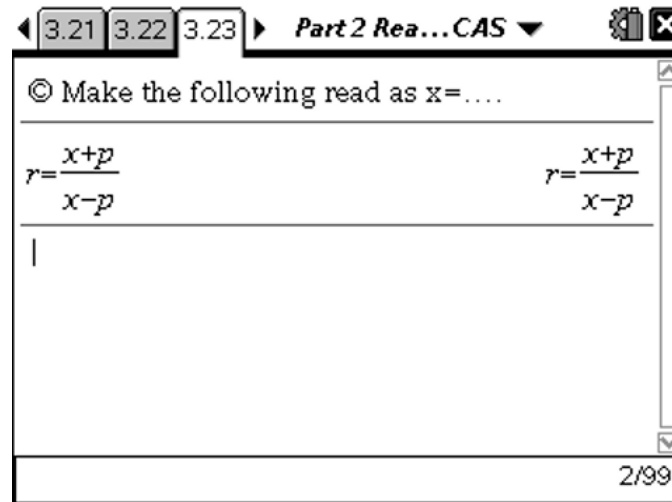
$$s(r-1) = 0$$

$$s = \frac{0}{r-1}$$

$$s = 0$$

?

hmm!



 Video of Elizabeth solving this.

www.CalculatorSoftware.co.uk/cas

Elizabeth's Jotter

$$18. r = \frac{x+p}{x-p}$$

$$r(x-p) = x+p$$

$$rx - pr = x+p$$

$$rx = x+p+pr \quad rx - pr - x = p$$

$$r(x-p) = x+p$$

~~18. r = \frac{x+p}{x-p}~~

$$x(r+1) = p+pr$$

$$x = \frac{p+pr}{r+1}$$

$$x = \frac{p}{r+1} + \frac{pr}{r+1}$$

$$x = p + \frac{p}{r+1}$$

$$18. r = \frac{x+p}{x-p}$$

$$r(x-p) = x+p$$

$$rx - pr = x+p$$

$$rx - pr - x = p$$

$$rx - x = p + pr$$

$$x(r-1) = p+pr$$

$$x = \frac{p+pr}{r-1} \text{ agrees}$$

Elizabeth's Test Homework (2 weeks later)

$$14. \quad R = \frac{rs}{rts}$$

$$R(rts) = rs$$

$$Rrts = rs$$

$$sR = rs - RR$$

$$-rst \quad sR = -Rr$$

$$s(R-r) = -Rr$$

$$s = \frac{-Rr}{r-r}$$

] R-r?

Ghasi's Test Homework (2 weeks later)

$$S = 2\pi r(r+h)$$

$$S = 2 \cdot \pi \cdot r^2 + 2 \cdot \pi \cdot r \cdot h$$

$$S - 2 \cdot \pi \cdot r^2 = 2 \cdot \pi \cdot r \cdot h$$

$$\frac{S - 2 \cdot \pi \cdot r^2}{2 \cdot \pi \cdot r} = h \quad \checkmark$$

$$L = M(1+at)$$

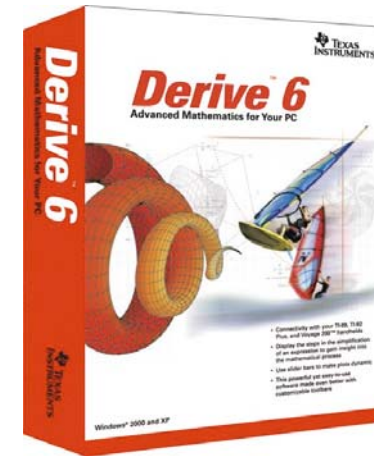
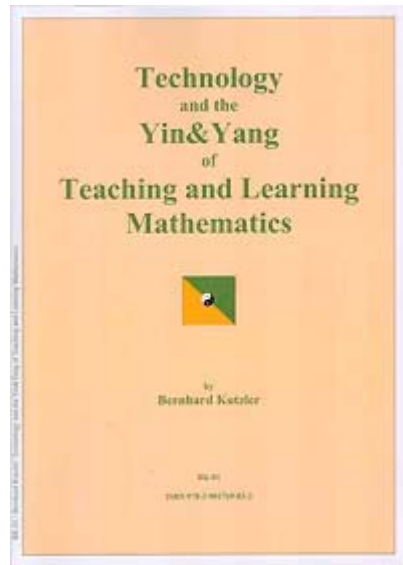
$$L = M + M \cdot a \cdot t$$

$$L - M = M \cdot a \cdot t$$

$$\frac{L - M}{M \cdot a} = t \quad \checkmark$$

My Experience versus Others' Experience

I met Bernhard Kutzler in November 2010....



'Educators Notes'

Powerful arguments and theory for promoting students to explore and learn from mistakes.

Introducing Surds with 15 & 17 Year Olds

Slide Title	File Status	Progress
1.1	*Unsaved	5/99
$\sqrt{4}$		2
$\sqrt{5}$		$\sqrt{5}$
$\sqrt{6}$		$\sqrt{6}$
$\sqrt{7}$		$\sqrt{7}$
$\sqrt{8}$		$2\sqrt{2}$

Unexpected Things with Integration

1.1 *Unsaved

$$\int \left(\frac{1}{x-3} + \frac{3}{(x-3)^2} \right) dx \quad \frac{(x-3) \cdot \ln(|x-3|) - x}{x-3}$$

$$\frac{(x-3) \cdot \ln(|x-3|)}{x-3} - \frac{x}{x-3} \quad \ln(|x-3|) - \frac{x}{x-3}$$

$$\text{expand} \left(\ln(|x-3|) - \frac{x}{x-3} \right) \quad \ln(|x-3|) - \frac{3}{x-3} - 1$$

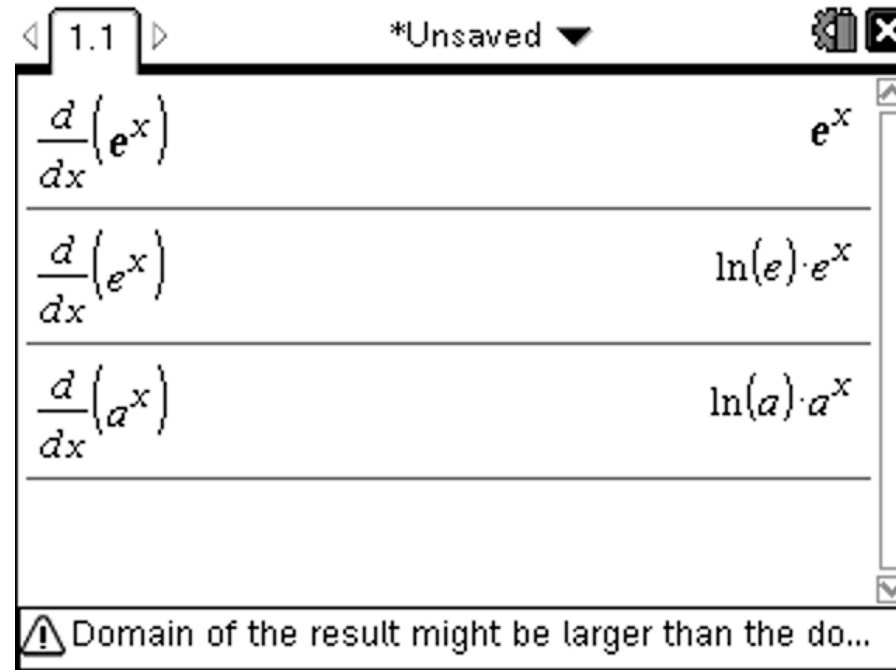
4/99

1.1 *Unsaved

$$\int \left(\frac{1}{x-3} \right) dx + \int \left(\frac{3}{(x-3)^2} \right) dx \quad \ln(|x-3|) - \frac{3}{x-3}$$

Should we request from TI that the constant of integration is included in the output from indefinite integrals?

Typos with Differentiation



The screenshot shows a software window titled "1.1" and "*Unsaved". It displays three rows of differentiation results:

$\frac{d}{dx}(e^x)$	e^x
$\frac{d}{dx}(e^x)$	$\ln(e) \cdot e^x$
$\frac{d}{dx}(a^x)$	$\ln(a) \cdot a^x$

At the bottom, a warning message reads: "Domain of the result might be larger than the do..."

When is e the same as \mathbf{e} ?

π is not treated like a Constant or a Variable



Where to Next?

- Write more CAS-specific activities for each topic that I come to teach, to allow students to explore and discover on their own.
- Re-read all the resources I collated from past conferences. I already see their role much better now, from 'the other side'.
- Recommend that schools buy Nspire CAS for their class-sets, as they'll never go into exam halls. They are **most** effective in the classroom where the learning happens.



Only 5 in the whole world!

Suggested minimum
donation of \$10.....

All proceeds to



Want More of Kilts & CAS?

www.CalculatorSoftware.co.uk/cas

Want More from Me?



**Session 330
"Two Minute Treasures"**

**Tomorrow.
In this same room.
10:00-11:00**

